Optical Differences Between Telescopes and Microscopes

Robert R. Pavlis, Girard, Kansas USA

Microscopes and telescopes are optical instruments that are designed to permit observation of objects and details of objects that are impossible to observe with the unaided eye. The term magnification as applied to telescopes refers to the degree of apparent increase of linear angular dimensions when an object is observed through the instrument. Magnification is defined in a similar manner for microscopes, but an observation distance needs to be supplied. The standard for this distance has been set for decades at 250 millimetres. Both types of instruments require an objective to form an image that is magnified by an eyepiece (ocular.) The formula that describes the relationship between objective focal length and the distances between the image plane and the object plane is one of the standard equations of optics:

$$\frac{1}{f_0} = \frac{1}{f_1} + \frac{1}{f_2}$$

where f_0 is the focal length of the objective, f_1 is the distance between it and the object being examined, and f_2 is the distance between the objective and the image plane presented to the ocular. There is one dramatic difference between the two types of instruments! f_1 is much smaller than f_2 in a telescope, whilst in a microscope this is reversed! For astronomical telescopes f_1 is usually light years, and in microscopes it is, at most, a few millimetres!

The optical design problems are dramatically different for the two types of instruments. The primary purpose of a microscope is to provide an enlarged image with as high a resolution as possible. The resolution of a microscope is limited by the numerical aperture of the objective and the wavelength of light used for the observation. This means the objective needs to have a very large "f-number"–that is the aperture needs to be very large relative to focal length.

Microscopes

First let us consider microscopes. Human eyes are sensitive to radiation having wavelengths between about 390 and 780 nm. Optical microscopes are designed to utilise these wavelengths to produce magnified images of objects. The wave nature of light restricts the resolution of microscopes. Microscope objectives take in a cone of light. The angle of the most divergent rays that can pass into an objective is called its angular aperture, often abbreviated "aa". The index of refraction is the ratio of the velocity of light in vacuum to that in an optical material. The numerical aperture of an objective is defined thus: $na = n \cdot \sin(\frac{aa}{2})$ The numerical aperture of a lens thus cannot exceed the index of refraction of the medium between the object being observed and the objective since the sin cannot exceed 1.0!

Mathematical analysis of light passing through a circular aperture shows that the minimum distance between resolvable points is given by:

$$d = \frac{j_{1,2} \cdot \lambda}{2 \cdot \pi \cdot na}$$

Where d is the distance between resolved points, $j_{1,2}$ is the first non-zero minimum value of the Bessel function J_1, λ is the wavelength of light, and na is numerical aperture. (The value of $j_{1,2}$ to 6 decimal places is 3.831705.)

One can write this formula with the constants evaluated to a single number:

$$d = \frac{1.22 \cdot \lambda}{2 \cdot na}$$

Note: Resolution figures quoted are usually for 546μ.

The optical glass used for the lower element of microscope objectives typically has n of about 1.51. By filling the space between the objective and the object with an oil of this index, one can achieve numerical apertures up about 1.4. Cedar wood oil was generally used for this purpose in the past; however, synthetic materials are more stable. The synthetics also are generally soluble in hexane, a less aggressive solvent than toluene or acetone, so there is less chance of damaging the objectives from solvent action. The highest power objectives used in microscopy always use this technique. Objectives that require immersion are usually marked with the words öl, oel, or oil. Note that immersion lenses require that the entire object being observed must be embedded in a high index of refraction material. One must not place a cover slip over a dry microscope slide and place oil on top of the cover slip without filling the space below the cover slip with oil or mountant! It is perfectly okay to dispense with the cover slip entirely and put the oil directly on the slide.

Microscope objectives are always marked with a power. For "finite optics" systems:

$$\boldsymbol{M_o = \frac{\Delta}{f_0}}$$

Where $\boldsymbol{M_o}$ is objective magnification, $\boldsymbol{\Delta}$ is optical tube length, and $\boldsymbol{f_0}$ is the distance from the object plane to the optical centre of the objective. For "infinity optics" systems $\boldsymbol{f_0}$ is the actual focal length of the objective. In these systems $\boldsymbol{\Delta}$ is the focal length of the tube lens. $\boldsymbol{\Delta}$ is almost always 160 mm for finite optics microscopes, however, Leitz manufactured many microscopes utilising $\boldsymbol{\Delta}$=170mm. $\boldsymbol{\Delta}$ varies from manufacturer to manufacturer with "infinity optics" systems. (This variation is probably deliberate on the part of manufacturers–they want purchasers of their microscopes to be forced to purchase their objectives!)

The index of refraction of glasses varies with wavelength, therefore a simple lens will only be in focus for one wavelength at a time causing what is called "chromatic aberration." Furthermore, with simple spherical surface lenses, spherical aberration causes objects not at the centre of field to become blurred. The solution to both of these problems is to utilise complex multi element lenses. By using two types of glass with different index of refraction and dispersion, one can produce achromats–lenses corrected to bring two wavelengths to focus at the same point and that are corrected for spherical aberration at one.

By using three or more types of lens material all with different indices of refraction and dispersion, it is possible to produce apochromats–lenses corrected to bring three or more wavelengths to focus at the same point, and that are corrected for spherical aberration in two more more.

The term semi-apochromat or fluorite objective generally is corrected for chromatic aberration in three wavelengths and spherical aberration in one. Although calcium fluoride (fluorite) is often used as one of the lens elements in both apochromats and semi-apochromates, the term "fluorite" generally refers to semi-apochromatic objectives.

For most chemical and biological uses of microscopes the sample being studied is best covered with a thin piece of glass, a so called "cover slip". The cover slip introduces severe spherical aberration when used with short focal length "dry" lenses with high numerical aperture lenses unless this lens is deliberately manufactured to compensate for the presence of the cover slip. Normal microscope objectives are manufactured to make this compensation and the thickness of the cover slip is marked on the housing of the lens. These corrections are virtually always made for 0.17 mm thick cover slips. Some "high end" objectives have adjustment collars that permit correcting for different thickness of cover slips.

Metallurgical objectives are designed to be used without a cover slip. Using high power objectives of this type with a cover slip will produce a poor image. Using a normal objective without one will produce an image just as poor! Immersion lenses obviously have no such problems, since cedar oil, the lower element of the lens, and cover slips all have the same index of refraction.

Metallurgical objectives generally have a system to illuminate the specimen from above built into the objectives. Optical considerations generally make it impossible to produce this type of objective with the standard DIN diameter.

Some manufacturers make special lenses for water immersion. These generally are designed for examining living specimens. They come in two varieties, those designed for use with cover slips, and those designed for direct immersion in the water solution holding the specimen.

A few manufacturers make "hybrid" objectives–ones that can used either with oil or water immersion and dry. These have an iris so that the field diaphragm can be adjusted for the medium.

The cone of light accepted by microscope objectives must, of necessity, be quite wide. (Otherwise the numerical aperture would be so small that the resolution would be very poor.) The cone of light accepted by microscope oculars, on the other hand, is rather narrow. Because of this spherical aberration in microscope oculars is minimal, thus the oculars can be quite simple with essentially no loss of resolution.

Finite tube length objectives generally require special oculars when used with apochromatic and flat field objectives because these objectives normally are designed to have “chromatic difference of magnification.” These special oculars are called compensating oculars. Many manufacturers fail to place any identifying marks on them, because they can be identified by simply looking at the field stop whilst holding them up to light. If the edge of the field stop appear reddish or yellowish it is compensating, if it appear bluish it is an ordinary one. Images will be seriously degraded by using compensating oculars with lenses that are designed not to use them. (And of course the reverse is true as well.) Occasionally there are problems using compensating oculars from one manufacturer with objectives from another.

Microscope manufacturers have agreed for over 100 years that the overall magnification of objects is to be expressed in terms of projecting the image 250 millimetres from the observers eye.

Thus the power stamped on oculars is 250/(focal length). The most useful ocular power is probably a 10X. Some people have a tendency to forget that quality oculars are important!

The diameter of the field stop in an ocular is called the “field number.” The actual linear measurement physical diameter of the field of observation is given by:

$$D = \frac{field \quad number}{M_o}$$

(M_o is OBJECTIVE magnification.)

Ordinary microscope oculars have an outside diameter of 23 millimetres, and, of course, inside diameter must be somewhat less than that, and there needs to be a retaining ring to hold the lenses in the ocular housing. This limits the field number to being less than about 21 millimetres. Some microscopes such as the Leitz Orthoplan have been built to require larger diameter oculars, almost always 30mm. This permits much wider angle observation.

Manufacturers often fail to specify the field number. It is a good idea to know this number to make it easier to estimate the size of objects observed under the microscope. When one is walking in the forest and encounters an animal one immediately recognises whether an animal seen walking in the forest is about the size of an elephant or the size of a mouse. Knowing the field number helps maintain similar prospective when viewing microscope objects!

When manufacturers do not provide this number, one can obtain it in a few seconds with the aid of a stage micrometer.

Note that there is also a simple relationship between apparent angular field of view and field number:

$$\theta = 2 \cdot atan\left(\frac{field \quad number \cdot M_e}{500}\right)$$

Where M_e is EYEPIECE magnification. The angular diameter of the cone of light from microscope objective is quite small. This means microscope oculars designs need to be corrected for images produced by objectives producing a wide cone of light. It is extremely difficult to make microscope objectives with really wide flat fields. Human eyes have resolution which results in 25mm focal length oculars–10X coming close to ideal with almost all objectives. Shorter focal lengths simply result in a less sharp image, and longer ones produce too narrow a field of view. Because low power objectives are made with smaller numerical aperture, they provide higher contrast than high power ones, microscope users tend to change objectives rather than oculars to change magnification.

Telescopes

TELESCOPES have many features and formulae in common with microscopes, but the similarities are often masked by differences in notation. The wave nature of light limits magnification of telescopes just as it does with microscopes.

$$\theta = \frac{j_{1,2} \cdot \lambda}{\pi \cdot D} = \frac{1.22 \cdot \lambda}{D}$$

where $j_{1,2}$ has the same value, 1.22, as it had in microscopes, where λ is again wavelength, where D is he diameter of the telescope objective, and where θ is angular resolution in radians.

With telescopes a larger objective serves two functions: (1) it increases resolution, and (2) it increases the amount of light gathered by the objective. Except for planetary observation, the latter function is usually far more important.

Telescope objectives thus need to be as large as practical for the application. Large objectives require enormous amounts of expensive glass, and the amount of glass varies with approximately the cube of the diameter! The mass of glass quickly becomes so great that it sags under its own weight, deforming its figure. It has proven impossible to make telescope objective lenses beyond about one metre in diameter! A very large amount of labour is required to grind and polish even small telescope lenses. Two elements made from different types of glass are required to produce an "achromatic" lens in which two colours come to focus at the same point because index of refraction varies with wavelength. Since each lens element has two sides, achromatic telescope lenses require grinding and polishing to great precision four optical surfaces. An achromatic lens corrected for two colours produces a fairly good image, but only TWO wavelengths are corrected really well. Objects, especially stars, still tend to show colour fringing. To reduce this require making apochromatic telescope objectives. These are designed for three wavelengths to come to focus at the same point–but this also requires three kinds of glass with different optical properties–and glass for this tends to be very expensive. An apochromatic objective of even 150 millimetres diameter is extremely expensive. (Modern formulations of glass have made it possible to make lenses with only two elements that have colour corrections that are nearly as good as with three.)

Large telescopes do not use objective lenses at all–instead they use mirrors. Mirrors completely eliminate colour fringing caused by using glass lenses because the light is reflected rather than refracted. The optical qualities of the mirror material do not matter at all! Furthermore mirrors only have one surface to grind and polish, and they can be supported by rigid structures behind them to prevent their sagging under their own weight. However, a mirror must have a parabolic figure to focus an object at infinity or there will be serious spherical aberration. Off axis images quickly develop "coma" so that stars off axis look like little comets. The large parabolic mirrors used in large observatories have a maximum field of view of much less than one degree because of this.

Furthermore, the mirror reflects the image forward, this requires a mirror in the light path to reflect the beam either off to the side (Newtonian telescope), or back toward an aperture in the mirror (Cassegrain telescope.) This secondary mirror reduces the light reaching the primary mirror, and it also introduces optical artifacts.

It is possible to reduce the off axis image problem by placing a special lens in front of a primary mirror with a spherical surface. A common way to do this is to utilise a complex lens with a fourth order curve on it, this is the famous Schmidt camera. In its original design it cannot be used visually, though modifications make it possible to make Schmidt-Cassegrain telescopes. A great many of these are sold to amateur astronomers.

Another solution to this problem is to use a strongly curved meniscus lens in front of the mirror. These lenses are expensive to produce, but the results are excellent, this is the Maksutov telescope. They are popular for small aperture instruments, but the high cost of production of the meniscus lens precludes using this design for large telescopes.

Because telescope objectives are huge and always expensive, the only practical way to change magnification is to change the eyepiece. Telescope eyepieces need to be optically more complex than the ones used in microscopes because they accept a relatively wide cone of light from the objective compared to microscopes. The magnification of a telescope is given by:

$$M = \frac{f_{objective}}{f_{ocular}}$$

The resolution of human eyes is such that under ideal atmospheric conditions magnifications greater than about 2X per millimetre are superfluous–the wave nature of light creates the same problem it did with microscopes. However, there is another serious problem associated with the structure of human eyes. The diameter of the iris of most individuals is around 7 millimetres. This means that the cone of light entering the eye from the telescope must be smaller than this. What this means is that the MINIMUM magnification of a telescope used for visual use must be greater than $\frac{D_{objective}}{D_{iris}}$. With large telescopes the minimum magnification is thus very high. For a 5 metre mirror, the minimum magnification is about 750! Thus large telescopes are typically not used visually.

A great many telescope types of telescope oculars are available. Almost all telescopes use standardised oculars that are either 51 or 31.75 mm in diameter. The term "field number" seldom is used by telescope manufacturers or users, but it has the same significance as it has in microscope objectives. A large fraction of recently manufactured small and medium sized telescopes utilise 51mm diameter oculars. This limits the field number to about 48 millimetres. Astronomers, particularly amateur ones, like oculars with very large apparent fields of views. With a 51mm diameter eyepiece the maximum possible apparent angular field of view that an optical designer can produce with this limitation is given by:

$$\boldsymbol{\theta = 2 \cdot atan\left(\frac{24}{F}\right)}$$

where $\boldsymbol{\theta}$ is the maximum angle of view possible and F is the focal length of the eyepiece. 100 degree eyepieces are available. These are thus limited to focal lengths less than about 20 millimetres. There are available a few 55 mm focal length eyepieces in 51 mm mounts. The maximum apparent field of view of these cannot exceed about 47 degrees!

It seems a bit odd to telescope users that microscope users do not designate microscope eyepieces and objectives by focal length rather than by the rather artificial convention and assumption that overall magnification should be given in terms of amount of increase in angular size at 250 millimetres! Microscope users generally realise that their convention simplifies determining overall magnification, especially since a large fraction of microscope eyepieces are 10X (25mm focal length).